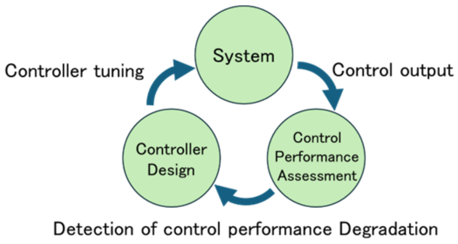
Design of a Performance-Driven PID Control System Based on an Extended MV-Index

■Yuki Ohtsu ■Takuya Kinoshita ■Toru Yamamoto

Graduate School of Advanced Science and Engineering Hiroshima University

1.Introduction

Performance-Driven PID control system



◎ Adaptable to system changes
(e.g., aging, changes in environmental conditions).

Control performance Assessment

■ MV-index (Minimum Variance index)

$$J = E[e(t)^2]$$

$$e(t) = r(t) - y(t)$$

■ EMV-index (Extended Minimum Variance index)

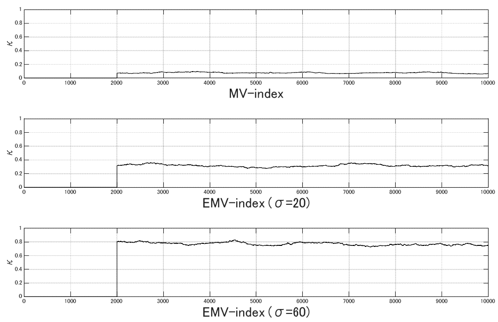
$$J = E[\phi(t)^2]$$

$$\phi(t) = P(z^{-1})y(t) - P(1)r(t-d-1)$$

$$P(z^{-1}) = 1 + p_1 z^{-1} + p_2 z^{-2}$$

$$\begin{cases} p_1 = -2e^{-2\rho} \\ p_2 = e^{-4\rho} \\ \rho = \dfrac{T_s}{\sigma} \end{cases}$$

σ: rise−time
T_s: sampling interval



MV-index : refer to the reference value $r(t)$

EMV-index : refer to the reference model output

2.Proposed Method

■ EMV-index

System: $A(z^{-1})y(t) = z^{-(d+1)}B(z^{-1})u(t) + \dfrac{\xi(t)}{\Delta}$

Evaluation criterion: $J = E[\phi^2(t+d+1)]$

$$\phi(t+d+1) = P(z^{-1})y(t+d+1) - P(1)r(t)$$

Diophantine equation: $P(z^{-1}) = \Delta A(z^{-1})E(z^{-1}) + z^{-(d+1)}F(z^{-1})$

$$\kappa = \frac{J_{\min}}{J_{\min} + J_0} \qquad \begin{cases} J_0 = E[(S(z^{-1})\xi(t-d-1))^2] \\ J_{\min} = E\left[(E(z^{-1})\xi(t))^2\right] \\ S(z^{-1}) = \dfrac{F(z^{-1}) - B(z^{-1})C(z^{-1})E(z^{-1})}{T(z^{-1})} \end{cases}$$

Computing $J_{\min}$ requires identifying the parameters of the polynomial $E(z^{-1})$

▼

using autoregressive (AR) model

$$\tilde{\boldsymbol{\Phi}}(t) = \boldsymbol{X}(t)\boldsymbol{\alpha}(t) + \boldsymbol{\Xi}(t)$$

$$\boldsymbol{\alpha}(t) = \left(\boldsymbol{X}(t)^T\boldsymbol{X}(t)\right)^{-1}\boldsymbol{X}(t)^{\mathrm{T}}\tilde{\boldsymbol{\Phi}}(t)$$

▼

$$\kappa(t) = \frac{J_{min}}{J_{min} + J_0} = \frac{\boldsymbol{\Xi}(t)^T\boldsymbol{\Xi}(t)}{\tilde{\boldsymbol{\Phi}}(t)^{\mathrm{T}}\tilde{\boldsymbol{\Phi}}(t)} = \frac{(\tilde{\boldsymbol{\Phi}}(t) - \boldsymbol{X}(t)\boldsymbol{\alpha}(t))^T(\tilde{\boldsymbol{\Phi}}(t) - \boldsymbol{X}(t)\boldsymbol{\alpha}(t))}{\tilde{\boldsymbol{\Phi}}(t)^{\mathrm{T}}\tilde{\boldsymbol{\Phi}}(t)}$$

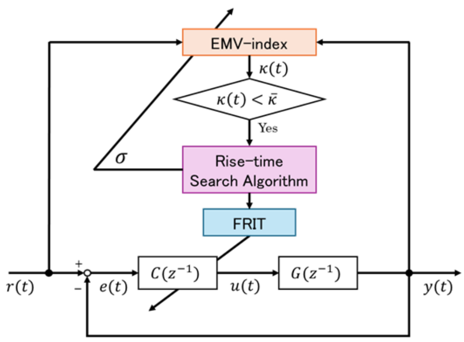
$$\tilde{\phi}(t) = \phi(t) - \bar{\phi}$$

$$\tilde{\boldsymbol{\Phi}}(t) := [\tilde{\phi}(t), \tilde{\phi}(t-1), \cdots, \tilde{\phi}(t-N+1)]^T$$

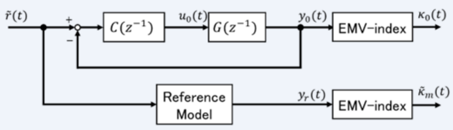
$$\boldsymbol{\alpha}(t) := [\alpha_1, \alpha_2, \cdots, \alpha_M]^T$$

$$\boldsymbol{\Xi}(t) := [\epsilon(t), \epsilon(t-1), \cdots, \epsilon(t-N+1)]^T$$

$$\boldsymbol{X}(t) := \begin{bmatrix} \phi(t-d-1) & \cdots & \phi(t-d-M) \\ \phi(t-d-2) & \cdots & \phi(t-d-M-1) \\ \vdots & \ddots & \vdots \\ \phi(t-d-N) & \cdots & \phi(t-d-M-N+1) \end{bmatrix}$$



■ FRIT
(Fictitious Reference Iterative tuning)



1. Obtain input-output data $u_0(t)$ and $y_0(t)$.
2. Calculate $\kappa_0(t)$ by using $y_0(t)$.
3. Calculate $y_r(t)$ and $\tilde{\kappa}_m(t)$ by using $u_0(t)$ and $y_0(t)$.
4. Optimize PID parameters of $C(z^{-1})$ to minimize the following equation

$$J = |\kappa_0 - \tilde{\kappa}_m|$$

■ Rise-time search algorithm

・The control performance index κ' is calculated using an adjustable reference model rise-time σ'.

・The optimal rise-time is determined so that κ is maximized based on the minimization of the following cost function J_σ.

$$J_\sigma = |1 - \kappa'|$$

3.Numerical Example

■ System

$$G(s) = \frac{10}{1 + Ts}e^{-8s} \qquad \begin{cases} T = 100 \ (t \le 5000) \\ T = 60 \quad (t > 5000) \end{cases}$$

■ User-specified parameters

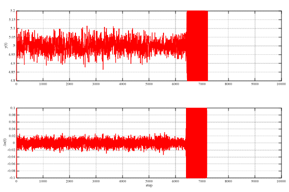
Variable	Value	Description
r	5	Reference signal
T_s	10	Sampling time
σ	54	Coefficient related to rise-time
N	500	Number of samples
M	20	Order of autoregressive parameters
$\bar{\kappa}$	0.75	Threshold of EMV-index
K_P	0.43	Initial P gain
K_I	0.11	Initial I gain
K_D	0.01	Initial D gain

■ previous method (Red)

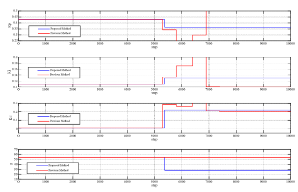
The control performance index remains below the threshold

▼

Over-tuning of PID parameters



Control result by using the previous method



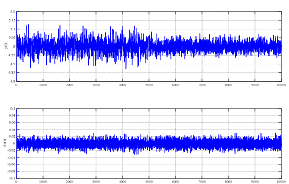
Trajectories of PID parameters and σ

■ proposed method (Blue)

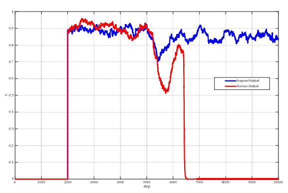
The update of the reference model allows the control performance index to exceed the threshold.

▼

Prevent over-tuning PID parameters



Control result by using the proposed method



Trajectory of κ

References

[1]: T.Yamamoto, Y.Ohnishi and S.L.Shah : Design of a Performance-Adaptive Proportional-Integral –Derivative Controller for Stochastic Systems, Institution of Mechanical Engineering, Part-I, Journal of Systems and Control Engineering, Vol.222, pp.691-699 (2008)
[2]: T.Kinoshita, Y.Ohnishi, T,Yamamoto and S.L.Shah : Design of a Data-Oriented Performance Driven Control System based on the Generalized Minimum Variance Control Law, Industrial ¥& Engineering Chemistry Research, Vol.58, No.26, pp.11440-11451 (2019)
[3]: S.Wakitani, T.Yamamoto and T.Sato : Design of a Performance-driven One-parameter Tuning Controller, International Journal of Control, Automation and Systems, Vol.18, No.6, pp.1453-1464 (2020)
[4]: T.J. Harris : Assessment of Closed Loop Performance, Canadian Journal of Chemical Engineering, Vol.67, No.5, pp.856-861 (1989)
[5]: B.Huang, andS.L. Shah : Performance Assessment of Control Loops: Theory and Applications; Springer-Verlag: London (1999)