Design of a Performance-Driven PID Control System based on an Extended MV-Index

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***Abstract*—The Performance-driven PID (Pd-PID) control method, which utilizes the minimum variance control index (MV-Index), has garnered significant attention. In Pd-PID, a deadbeat control system is always considered optimal for evaluation, regardless of the system's response speed. As a result, systems with slower responses receive lower evaluations, even when good control performance can be obtained. Furthermore, the Pd-PID method continuously updates PID parameters to improve control performance evaluation, which can ultimately lead to system instability. This paper addresses this issue and proposes a new control performance evaluation index to replace the MV-Index.**

***Index Terms*—Performance-driven PID control, Control performance evaluation, Minimum variance control, Data-driven control**

I. Introduction

It is known that the characteristics of many industrial systems change due to variations in environmental and operational conditions, as well as aging deterioration. Even under such circumstances, maintaining the desired control performance remains essential. To address this issue, performance-driven PID control [1]- [3], which integrates control performance assessment with control system design, has garnered significant attention.

As a method for control performance assessment used in performance-driven PID control, the minimum variance control index (MV-index) [4] [5] which focuses on the variance of the output, has been proposed. However, the MV-index is based on deadbeat control, where the output follows the setpoint after a time-delay. Since deadbeat control requires excessive control input, it is not preferable for practical systems.

To overcome this issue, the extended minimum variance control index (EMV-index) [3], which takes a reference trajectory into account, has been proposed. The EMV-index ensures that the output follows the reference trajectory, enabling performance assessment that also considers the response speed of the control system. However, the assessment value can vary significantly depending on the setting of the reference trajectory, making the proper design of the reference model crucial.

This paper focuses on the dependence of the control performance assessment using the EMV-index on the rise-time of the reference model and explores a design method for performance-driven control systems based on the EMV-index. Specifically, a method is proposed to detect system variations using the EMV-index and construct a new reference model that corresponds to the system characteristics after variation. Furthermore, based on the constructed reference model, control parameters are adjusted using FRIT. Finally, numerical simulations are conducted to verify the effectiveness of the proposed method.

II. Control Performance Evaluation

A. System Model

The following discrete-time system is considered:

$$A(z^{-1})y(t) = z^{-(d+1)}B(z^{-1})u(t) + \frac{\xi(t)}{\Delta} \tag{1}$$

In Eq.(1), Δ is the difference operator ($\Delta := 1 - z^{-1}$); $\xi(t)$ is the noise; and $d(\geq 0)$, the time-delay. In addition, $A(z^{-1})$, $B(z^{-1})$ are given as follows:

$$\begin{cases} A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_n z^{-n} \\ B(z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_m z^{-m} \end{cases} \tag{2}$$

In Eqs.(2), n, m denote the orders of $A(z^{-1})$, $B(z^{-1})$. The system in Eq.(1) is assumed to satisfy the following assumptions.

[Assumptions]

1) n and m are given, and $A(z^{-1})$ and $B(z^{-1})$ are irreducible.
2) $B(z^{-1})$ is a polynomial that is asymptotically stable.
3) $\xi(t)$ is Gaussian white noise that satisfies the following conditions.

$$\begin{cases} E[\xi(t)] = 0 \\ E[\xi^2(t)] = \sigma_\xi^2 \\ E[\xi(t)\xi(t+\tau)] = 0 \qquad (\tau \neq 0), \end{cases} \tag{3}$$

where $E[\cdot]$ denotes the expectation, and σ_ξ represents the standard deviation of $\xi(t)$.

4) The setpoint $r(t)$ is constant.

In this paper, the PID controller is introduced as $C(z^{-1})/\Delta$ in the following equation:

$$u(t) = \frac{C(z^{-1})}{\Delta}(r(t) - y(t)) \tag{4}$$

B. Extended Minimum Variance Control Index

The extended minimum variance control law for the system described by Eq.(1) can be derived by minimizing the following cost function:

$$J = E[\phi^2(t+d+1)], \tag{5}$$

where $\phi(t+d+1)$ is defined as

$$\phi(t+d+1) := P(z^{-1})y(t+d+1) - P(1)r(t) \tag{6}$$

Next, consider the Diophantine equation given by Eq.(7):

$$P(z^{-1}) = \Delta A(z^{-1})E(z^{-1}) + z^{-(d+1)}F(z^{-1}) \tag{7}$$

$$\begin{cases} E(z^{-1}) = 1 + e_1 z^{-1} + \cdots + e_d z^{-d} \\ F(z^{-1}) = f_0 + f_1 z^{-1} + \cdots + f_n z^{-n} \end{cases} \tag{8}$$

Furthermore, $P(z^{-1})$ is a user-specified polynomial based on the reference model and is defined as follows:

$$P(z^{-1}) = 1 + p_1 z^{-1} + p_2 z^{-2} \tag{9}$$

$$\begin{cases} p_1 = -2e^{-2\rho} \\ p_2 = e^{-4\rho} \\ \rho = T_s/\sigma, \end{cases} \tag{10}$$

where T_s and σ denote the sampling time and rise-time. From Eqs.(1) and (4), the following equation is obtained:

$$y(t) = \frac{z^{-(d+1)}B(z^{-1})C(z^{-1})}{T(z^{-1})}r(t) + \frac{1}{T(z^{-1})}\xi(t), \tag{11}$$

where $T(z^{-1})$ is defined as in Eq.(12):

$$T(z^{-1}) := \Delta A(z^{-1}) + z^{-(d+1)}B(z^{-1})C(z^{-1}) \tag{12}$$

From Eqs.(6) and (11), $\phi(t+d+1)$ can be transformed as follows:

$$\begin{aligned} \phi(t+d+1) = {} & \frac{B(z^{-1})C(z^{-1})(P(z^{-1}) - z^{-(d+1)}P(1))}{T(z^{-1})}r(t) \\ & + \frac{\Delta A(z^{-1})P(1)}{T(z^{-1})}r(t) + \frac{P(z^{-1})}{T(z^{-1})}\xi(t+d+1) \end{aligned} \tag{13}$$

Since, by assumption, the setpoint remains constant during the time-delay, the first and second terms on the right-hand side of Eq.(13) become zero. Furthermore, multiplying both sides by $E(z^{-1})T(z^{-1})$ yields the following equation.

$$E(z^{-1})T(z^{-1})\phi(t+d+1) = E(z^{-1})P(z^{-1})\xi(t+d+1) \tag{14}$$

Next, from Eqs. (7) and (12), we obtain the following equation:

$$\begin{aligned} E(z^{-1})T(z^{-1}) = {} & P(z^{-1}) - z^{-(d+1)}F(z^{-1}) \\ & + z^{-(d+1)}B(z^{-1})C(z^{-1})E(z^{-1}) \end{aligned} \tag{15}$$

Finally, from Eqs. (14) and (15), the following equation can be obtained:

$$\phi(t+d+1) = E(z^{-1})\xi(t+d+1) + S(z^{-1})\xi(t), \tag{16}$$

where $S(z^{-1})$ is given by

$$S(z^{-1}) := \frac{F(z^{-1}) - B(z^{-1})C(z^{-1})E(z^{-1})}{T(z^{-1})} \tag{17}$$

From the above derivations, Eq.(5) can be rewritten as:

$$\begin{aligned} J &= E[(E(z^{-1})\xi(t+d+1) + S(z^{-1})\xi(t))^2] \\ &= J_{\text{min}} + J_0, \end{aligned} \tag{18}$$

where

$$J_{\text{min}} = E[(E(z^{-1})\xi(t+d+1))^2] \tag{19}$$

$$J_0 = E[(S(z^{-1})\xi(t))^2] \tag{20}$$

Here, when an optimal controller is employed, $J_0 = 0$ holds, In other words, the minimum variance is achieved when $J_0 = 0$. Based on Eq.(18), the EMV-index is defined by κ as follows:

$$\kappa := \frac{J_{\text{min}}}{J_{\text{min}} + J_0} = 1 - \frac{J_0}{J_{\text{min}} + J_0} \tag{21}$$

C. Calculation of EMV-index by AR model

It is necessary to solve the identity in Eq.(7) in order to obtain $E(z^{-1})$ for calculating J_{min} in Eq.(19). In this paper, κ is approximately computed from closed-loop data using an AR model [4] [5]. First, $\phi(t)$ is modeled by the AR model as

$$\phi(t) - \bar{\phi} = \epsilon(t) + \sum_{i=0}^{M} \alpha_i(\phi(t-d-i) - \bar{\phi}) \tag{22}$$

$$\epsilon(t) := E(z^{-1})\xi(t), \tag{23}$$

where $\bar{\phi}$ is the mean value of $\phi(t)$, α_i are the parameters of the auto-regressive model, and M is the order of α_i. α_i can be estimated using the least squares method as follows:

$$\tilde{\boldsymbol{\Phi}}(t) = \boldsymbol{X}(t)\boldsymbol{\alpha}(t) + \boldsymbol{\Xi}(t) \tag{24}$$

$$\tilde{\phi}(t) := \phi(t) - \bar{\phi} \tag{25}$$

$$\tilde{\boldsymbol{\Phi}}(t) := [\tilde{\phi}(t), \tilde{\phi}(t-1), \cdots, \tilde{\phi}(t-N+1)]^T \tag{26}$$

$$\boldsymbol{\alpha}(t) := [\alpha_1(t), \alpha_2(t), \cdots, \alpha_M(t)]^T \tag{27}$$

$$\boldsymbol{\Xi}(t) := [\epsilon(t), \epsilon(t-1), \cdots, \epsilon(t-N+1)]^T \tag{28}$$

$$\boldsymbol{X}(t) := \begin{bmatrix} \tilde{\phi}(t-d-1) & \cdots & \tilde{\phi}(t-d-M) \\ \tilde{\phi}(t-d-2) & \cdots & \tilde{\phi}(t-d-M-1) \\ \vdots & \ddots & \vdots \\ \tilde{\phi}(t-d-N) & \cdots & \tilde{\phi}(t-d-M-N+1) \end{bmatrix} \tag{29}$$

$$\boldsymbol{\alpha}(t) = (\boldsymbol{X}(t)^T\boldsymbol{X}(t))^{-1}\boldsymbol{X}(t)^T\tilde{\boldsymbol{\Phi}}(t) \tag{30}$$

In addition, N denotes the number of samples (or observations) of the closed-loop data. In order to accurately compute the EMV-index κ, it is desirable to select a sufficiently large value for N. However, increasing N also leads to higher computational complexity, and thus a trade-off between accuracy and computational efficiency must be considered. The EMV-index κ is defined as:

$$\kappa = \frac{(\tilde{\boldsymbol{\Phi}}(t) - \boldsymbol{X}(t)\boldsymbol{\alpha}(t))^T(\tilde{\boldsymbol{\Phi}}(t) - \boldsymbol{X}(t)\boldsymbol{\alpha}(t))}{\tilde{\boldsymbol{\Phi}}(t)^T\tilde{\boldsymbol{\Phi}}(t)} \tag{31}$$

D. Optimization of the Rise-time to Maximize κ

Since the EMV-index is significantly affected by the design of the reference model, it is necessary to redesign the reference model corresponding to the post-fluctuation system when system fluctuations occur. By using an adjustable rise-time, denoted by σ', reference-model-based polynomial $P'(z^{-1})$ can be designed based on Eqs.(9) and (10). Furthermore, by employing $P'(z^{-1})$ an adjustable control performance assessment index κ' can be derived following the same procedure as in Eq.(31). Next, the following cost function is considered:

$$J_\sigma = |1 - \kappa'| \tag{32}$$

Here, minimizing Eq.(32)) is equivalent to finding the rise-time of the reference model that brings κ closer to 1. Therefore, by determining σ_{opt} that minimizes Eq.(32), a reference model corresponding to the post-fluctuation system can be designed.

III. PID Tuning Using FRIT Based on Control Performance Evaluation

Fig.1 shows a block diagram of the proposed fictitious reference iterative tuning (FRIT) approach. Here, $u_0(t)$ and $, y_0(t)$ are a set of input-output data; $\tilde{r}(t)$ is the fictitious reference input; $y_r(t)$ is the fictitious reference output; $\kappa_0(t)$ and $\tilde{\kappa_m}(t)$ are denote the control performance assessment indices for $y_0(t)$ and $y_r(t)$, respectively.

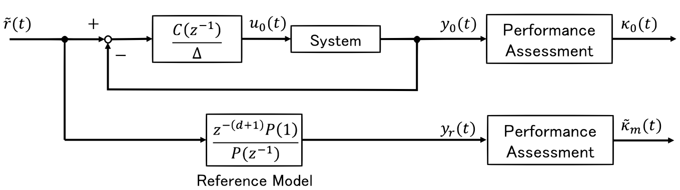

Fig. 1. Block Diagram of Pd-FRIT

From Eqs.(6) and (31), the control performance assessment index κ_0 can be calculated using $y_0(t)$. Furthermore, the fictitious reference input is obtained as follows:

$$\tilde{r}(t) = C(z^{-1})^{-1}\Delta u_0(t) + y_0(t) \tag{33}$$

The fictitious reference output $y_r(t)$ is given by:

$$y_r(t) = \frac{z^{-(d+1)}P(1)}{P(z^{-1})}\tilde{r}(t) \tag{34}$$

Thus, $\tilde{\kappa_m}(t)$ can be calculated as follows:

$$\tilde{\kappa_m} = \frac{(\tilde{\boldsymbol{\Psi}} - \tilde{\boldsymbol{X}}(t)\tilde{\boldsymbol{\alpha}}(t))^T(\tilde{\boldsymbol{\Psi}} - \tilde{\boldsymbol{X}}(t)\tilde{\boldsymbol{\alpha}}(t))}{\tilde{\boldsymbol{\Psi}}^T\tilde{\boldsymbol{\Psi}}}, \tag{35}$$

where each variable in Eq.(35) is defined as follows:

$$\tilde{\boldsymbol{\alpha}}(t) = (\tilde{\boldsymbol{X}}(t)^T\tilde{\boldsymbol{X}}(t))^{-1}\tilde{\boldsymbol{X}}(t)^T\tilde{\boldsymbol{\Psi}}(t) \tag{36}$$

$$\tilde{\psi}(t) := P(z^{-1})y_r(t) - P(1)r(t-d-1) \tag{37}$$

$$\tilde{\boldsymbol{\Psi}}(t) := [\tilde{\psi}(t), \tilde{\psi}(t-1), \cdots, \tilde{\psi}(t-N+1)]^T \tag{38}$$

$$\tilde{\boldsymbol{X}}(t) := \begin{bmatrix} \tilde{\psi}(t-d-1) & \cdots & \tilde{\psi}(t-d-M) \\ \tilde{\psi}(t-d-2) & \cdots & \tilde{\psi}(t-d-M-1) \\ \vdots & \ddots & \vdots \\ \tilde{\psi}(t-d-N) & \cdots & \tilde{\psi}(t-d-M-N+1) \end{bmatrix} \tag{39}$$

Finally, the control parameters of $C(z^{-1})$ are determined by minimizing the absolute error between κ_0 and $\tilde{\kappa_m}$.

IV. Numerical Example

The effectiveness of the proposed scheme is verified using a numerical example. The controlled object is represented by the following first order system with time-delay.

$$G(s) = \frac{10}{1+Ts}e^{-8s} \tag{40}$$

$$\begin{cases} T = 100 & (t <= 5000) \\ T = 60 & (t > 5000) \end{cases} \tag{41}$$

Here, t denotes the step index in discrete-time, and the Gaussian white noise $\xi(t)$ has a mean of 0 and a variance of 0.01^2. The user-specified parameters are determined as shown in Table.I. In particular, the values of N and M are selected to achieve a balance between the accuracy of the EMV-index and the computational cost. The initial rise time σ is set to half the sum of the system's time constant and time delay.

TABLE I
USER-SPECIFIED PARAMETERS

Variable	Value	Description
r	5	Reference signal
K_P	0.43	Initial P gain
K_I	0.11	Initial I gain
K_D	0.01	Initial D gain
σ	54	Coefficient related to rise time
N	500	Number of samples
M	20	Order of autoregressive parameters
κ	0.75	Threshold of EMV-index

The simulation results are shown in Figs.2 to 5. Specifically, Figs.2 and 3 presents the control results $\Delta u(t)$ and $y(t)$; Fig.4 illustrates the transitions of the PID parameters and the rise-time of the reference model; and Fig.5 shows the control performance assessment, $\kappa(t)$. Additionally, for comparison, results are provided the-rise time remains unchanged.

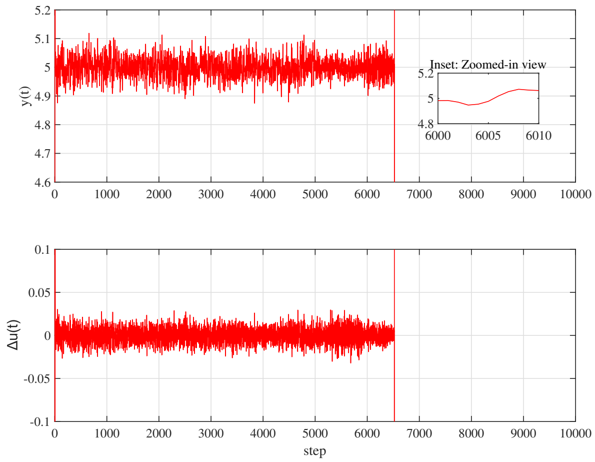

Fig. 2. control result by using the previous method

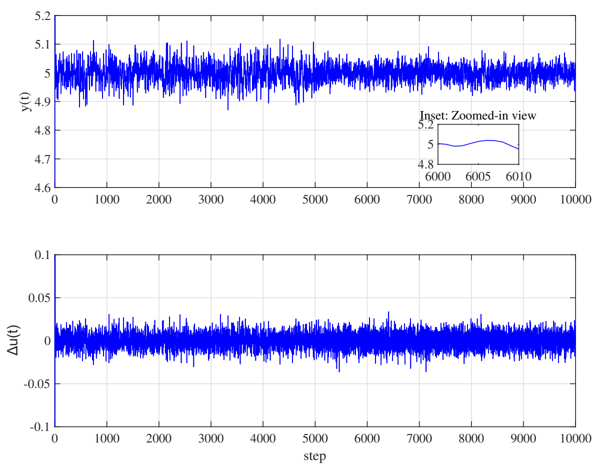

Fig. 3. control result by using the poposed method

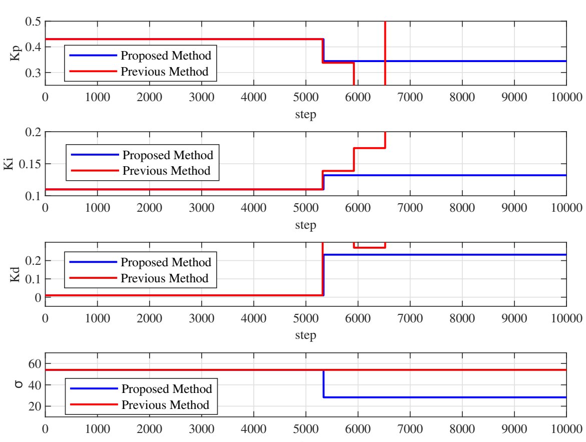

Fig. 4. Trajectories of PID parameters and σ corresponding to Figs.2 and 3

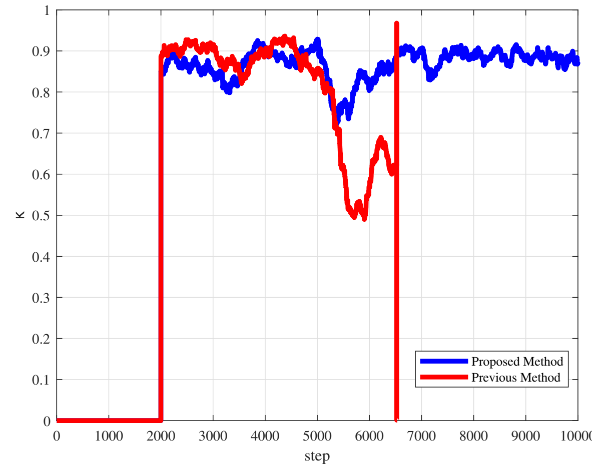

Fig. 5. Trajectory of κ corresponding to Figs.2 and 3

Figs.2 to 5 demonstrate that the previous method fails to meet the control performance assessment threshold even after executing FRIT. As a result, parameter tuning is repeatedly performed, which not only fails to improve the control response but ultimately leads to system instability. In contrast, the proposed method updates the rise-time, σ, of the reference model, effectively improving control performance and achieving a stable control response. It should be emphasized that although the time constant is changed at step 5000, there is a delay in the adjustment of the PID gains. This delay arises from the necessity of acquiring sufficient post-change data before applying FRIT reliably.

V. Conclusion

In this paper, a method has been proposed for updating the rise-time of the reference model in response to system fluctuations as a design approach for performance-driven control. The proposed method adaptively determines the rise-time of the reference model based on the system characteristics, demonstrating that a stable control system can be designed as a result. Future work will focus on experimental validation using physical systems and the extension of the proposed approach to cyber-physical system applications.

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